Duct Resonance and Its Effect on the Performance of High-Pressure Ratio Axisymmetric Ejectors

Adnan M. Abdel-Fattah* and Shane C. Favaloro†
Defense Science and Technology Organization, Melbourne, Australia

The effect of duct resonance on the performance of high-pressure ratio axisymmetric ejectors was investigated in a series of experiments involving analysis of the emitted noise and schlieren photography of the ejector duct flow. The thrust characteristics obtained with changing blowing pressure were found to contain irregularities, and these were found to be associated with certain tones of acoustic radiation which tuned to the transverse modes of the mixing duct. When varying the blowing pressure for a given ejector the discrete tone fundamental frequency showed a staging behavior, and the frequencies at the discontinuities separating the stages coincided with the calculated modes of the duct. For a given frequency discontinuity, the Helmholtz number (based on duct diameter), was found to be the same for all ejectors tested. For each family of ejectors having the same duct-length to diameter ratio, the blowing pressure at which certain discontinuities occurred varied linearly with duct to nozzle area ratio, and the thrust irregularities assumed similar linear relationships. Increasing the duct length greatly improved thrust augmentation in the relatively high-pressure range and shifted thrust irregularities and the corresponding frequency discontinuities to lower pressure ranges. The schlieren photographs showed that the primary jet spread, and its mixing with the entrained secondary stream, were evidently enhanced by duct resonance. The jet appeared to be surrounded by interspersed helical eddies and to be oscillating in a spinning mode.

Nomenclature

A_D	= ejector duct cross-sectional area
A_N	= nozzle exit cross-sectional area
c	= ambient speed of sound
d_*	= nozzle throat diameter
D	= diameter of the ejector duct
f	= frequency of the fundamental discrete tone
$f_{p,n}$	= natural resonant modes of the circular ejector duct
p,n	= number of nodal diameters and nodal circles respectively
F	= duct axial thrust
H_n	= Helmholtz number $= fD/c$
L	= length of the ejector duct
P_o	= jet stagnation pressure
P_a	= atmospheric pressure
P_l	= static pressure at the nozzle exit plane
$\nu_{p,n}$	= roots of Bessel functions

I. Introduction

UNSTEADY effects in various forms, when imposed on the mixing process between the primary jet and the entrained secondary stream in a confining ejector duct, can greatly enhance mixing and minimize the duct length required for complete energy and momentum transfer between the two streams. Prior work[1,2] has demonstrated that jets are susceptible to deformation by acoustic signals such as those generated by a loudspeaker. In the case of an incorrectly expanded supersonic free jet, intense acoustic signals are emitted from the jet due to the presence of the cellular shock structure which is aerodynamically created in the jet core to compensate for the pressure differences at the jet boundary. The shock associated noise can contain two components. The first is broadband in nature and the other discrete. The discrete tones of acoustic radiation, often called screech, which appear in the frequency spectrum as high-amplitude spikes, usually with harmonics, were first recognized and studied by Powell,[3,4] who proposed a simple feedback mechanism for their generation. In the case of a screeching jet in the presence of reflecting surfaces, intense acoustic signals are reflected back to the jet, resulting in its deformation and causing it to spread and decay. This was first demonstrated by Glass[5] who, by replacing the reflecting surfaces with sound absorbing materials, was able to interrupt the feedback loop and stabilize the supersonic jet. It was also shown by Nagel et al.[6] that screech could even be canceled with sound reflecting surfaces positioned upstream of the nozzle exit at a distance of one-quarter wave length of the fundamental discrete tone. The importance of reflecting surfaces near the nozzle exit of a nonvibrating jet and with externally generated pulses was demonstrated by Poldervaart et al.[7] They concluded that the eddy does not play the role in the feed back loop given in Powell's[3] model, and the oscillations were initiated by the pulse boundary layer interaction at the nozzle lip.

Discrete tones of acoustic radiation were also emitted from confined incorrectly expanded supersonic jets,[8–12] but most of these studies were performed to investigate the acoustic properties of supersonic ejector configurations driven by the need to explore means for jet noise suppression. All these studies were carried out with convergent nozzles in a relatively low range of jet stagnation pressures up to 4 atm. In a much higher stagnation pressure range (up to 20 atm), but moderate when compared with the range of the present investigation, Mamin et al.[13] reported on discrete tone emission from supersonic jets issuing from convergent divergent nozzles at the inlet of cylindrical ejector ducts. They found the frequency of the fundamental discrete tone in its variation with jet stagnation pressure to be discontinuous, and, at these discontinuities, the frequencies were equal to the first transversal modes of the ejector mixing duct. Quinn[14,15] was the first to investigate the interaction between the discrete tones and ejector performance. All his experiments were performed with one circular convergent

Received Feb. 25, 1987; presented as Paper 87-7055 at the AIAA 8th Symposium on Air-Breathing Engines, Cincinnati, OH, June 15–19, 1987; revision received Aug. 17, 1987.

*Senior Research Scientist, Aero Propulsion Division, Aeronautical Research Laboratories.
†Experimental Officer, Aeronautical Research Laboratories.

nozzle in a stagnation pressure range up to 6 atm at the inlet of a family of constant area ducts of one diameter (duct to nozzle area ratio) but with different duct lengths. He found the ejector performance to show some irregularities at certain stagnation pressure ratios, and these irregularities to coincide with discernable changes in the emitted discrete tone frequency. He showed this happened when the discrete tone frequency was tuned to some of the transverse modes of the ejector duct. With much higher pressures and area ratios, similar to those used in the present experiments, Fisher[16] also observed thrust irregularities in a range of ejector geometries which were somewhat more pronounced than those reported by Quinn.

As part of a broader investigation of the use of ejectors for improving the static thrust of rocket motors, the discrete tone frequencies and the corresponding ejector duct thrust characteristics obtained experimentally for several ejector configurations are presented and discussed in this paper. This is an extension to an earlier investigation[17] which demonstrated the profound effects of these tones on thrust augmentation. In addition to spectral analysis, the present investigation includes visualization of the duct flow field to reveal the effect of duct resonance on the behavior of the jet and on the entrainment process, and hence on thrust augmentation.

II. Experimental Apparatus

Tests were conducted using a family of five conical convergent-divergent nozzles of different size but with the same exit to throat area ratio of 4.03. The nozzle throat diameters were 5.56, 6.18, 7.75, 9.25, and 12.32 mm. These were tested with three constant cross-sectional area ducts of different diameters and lengths, such that the duct to nozzle exit area ratio varied between 9 and 176. Four values of duct length to diameter were tested, namely $L/D = 2.25$, 4.25, 6.25, and 8.75. Unheated air was supplied from storage vessels at room temperature over a regulated pressure range up to 60 atm.

The duct thrust variation as a function of jet stagnation pressure for different ejector configurations was obtained by a Mosley-Model 135 series, X-Y Plotter. This was continuously activated by the output of a pressure transducer connected to the jet stagnation pressure supply and an Interface, JNC SM-50 force cell connected to the ejector duct. The noise generated was measured with a Bruel & Kjaer 13.2 mm condenser microphone located at a fixed position 50 deg upstream from the nozzle exit in the horizontal plane containing the nozzle centerline, at a radius of 148 mm. The amplified microphone signal was recorded with a four-channel tape recorder. The recorded signal was analyzed and the noise spectrum plotted with a Spectral Dynamics Corporation model SD335 spectroscope real time analyzer. The jet flow field was photographed in the horizontal and vertical planes with a conventional schlieren system using a high-intensity spark of about 0.5 μs duration which was fast enough to freeze the high-speed jet flow field and associated emitted phenomena. A schematic diagram of the experimental apparatus is shown in Fig. 1.

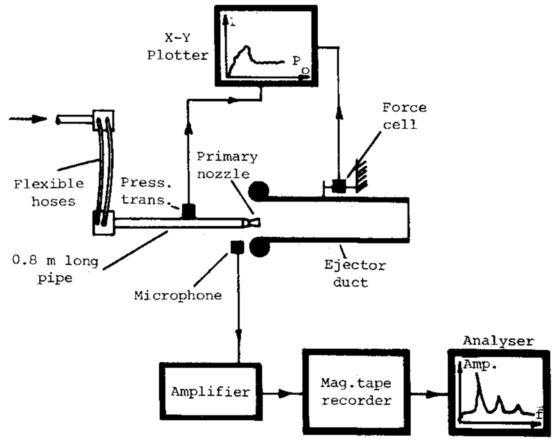

Fig. 1 Schematic diagram of the experimental apparatus.

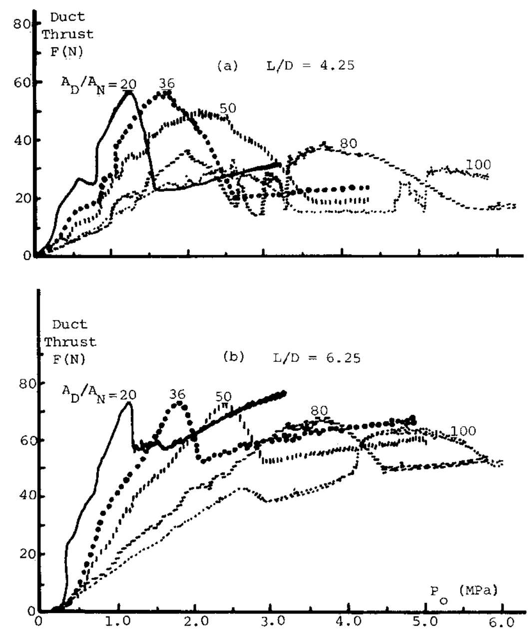

Fig. 2 Duct axial thrust for two families of ejectors of the same duct diameter but with different nozzle sizes. a) $L/D = 4.25$, b) $L/D = 6.25$.

III. Results and Discussion

Thrust Characteristics

Figure 2 shows the experimental duct thrust characteristics for two ejector families of fixed duct length to diameter ratio, $L/D = 4.25$ (Fig. 2a) and $L/D = 6.25$ (Fig. 2b). The characteristics were obtained for each family by continuously, but slowly, varying the stagnation pressure for different values of duct to nozzle area ratio. The time required to obtain each characteristic was on the average of ten minutes which was long enough to eliminate any possibility of the response times of the sensors to affect the result. Furthermore, these traces are repeatable, and no sign of hysteresis could be noticed, whether obtained with reducing or increasing the jet blowing pressure, and with the irregularities occurring at the corresponding blowing pressures.

Shown in Fig. 3 are the thrust characteristics for another two ejector families of fixed duct to nozzle area ratio, $A_D/A_N = 36$ (Fig. 3a) and $A_D/A_N = 80$ (Fig. 3b) with different L/D ratios. Included for comparison are curves calculated using fully compressible inviscid ejector theory, employing principles of conservation of mass, momentum and energy for the ejector flow between the injection and exit planes. The theory was idealized in the sense that: 1) The primary jet and entrained air flows at the inlet were uniform, parallel and coaxial with the duct, 2) The flow at the exit was fully mixed and uniform, 3) Skin friction, inlet and nozzle losses were ignored. Thrust values calculated using such theory are, of course, independent of duct length. Theoretical values of duct thrust were separated out from total ejector thrust by subtracting the theoretical isentropic thrust of the nozzle used in the experiments.

Figure 3 shows that the levels of thrust that can be achieved experimentally are always lower than the corresponding theoretical values. In the simple ejector configurations used in the present investigation this can be due to either skin friction, inlet efficiency, nozzle thrust efficiency, incomplete mixing between the primary and entrained secondary streams, or any combination of these effects. Increasing the mixing duct length for an ejector of given A_D/A_N, as expected, improves the levels of

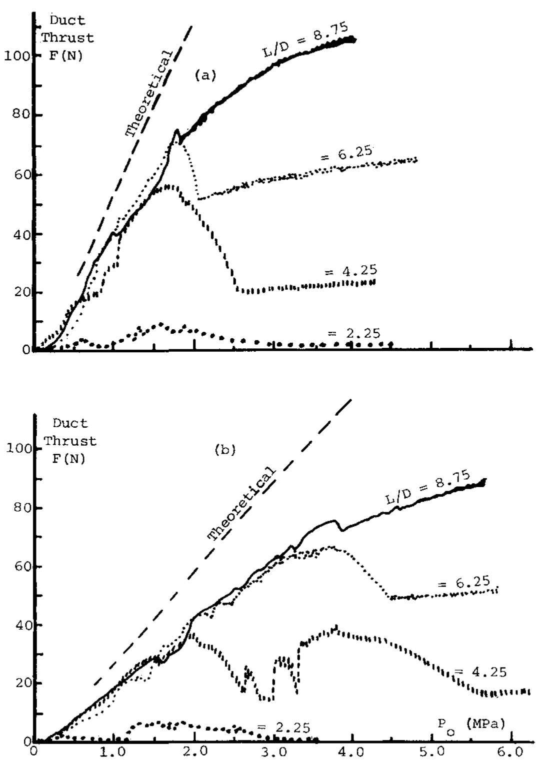

Fig. 3 Duct axial thrust for two families of ejectors of the same duct diameter but with different duct lengths. a) $A_D/A_N = 36$, b) $A_D/A_N = 80$.

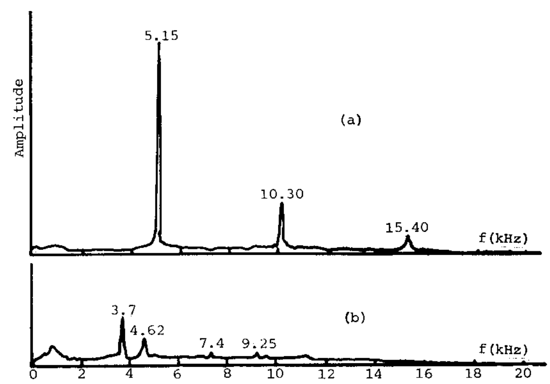

Fig. 4 Ejector noise spectra obtained with $A_D/A_N = 20$ at two different jet stagnation pressure ratios. a) $P_o/P_1 = 9.66$, b) $P_o/P_1 = 16.12$.

thrust augmentation by providing the two streams in the duct with more time to mix and exchange their momentum and energy. This is clearly demonstrated in Fig. 3, where most of the improvement appears to be in the relatively high range of jet stagnation pressure. In the low pressure range, the thrust characteristics for different duct lengths appear to collapse onto a single characteristic which defines the limiting maximum thrust at a given pressure which can be achieved experimentally with any duct length. This limiting experimental curve defines the fully mixed flow condition at the mixing duct exit. The difference in thrust levels defined by this curve and the theoretical one are due to other effects such as friction, inlet losses, and nozzle efficiency.

In both Figs. 2 and 3 it is evident that the thrust characteristic obtained for each ejector configuration displayed irregularities that cannot be predicted or explained by the idealized ejector theory, and which reflect corresponding variations in the levels of mixing between the two streams in the ejector duct. These irregularities were more pronounced with short ejectors and with large duct to nozzle area ratios.

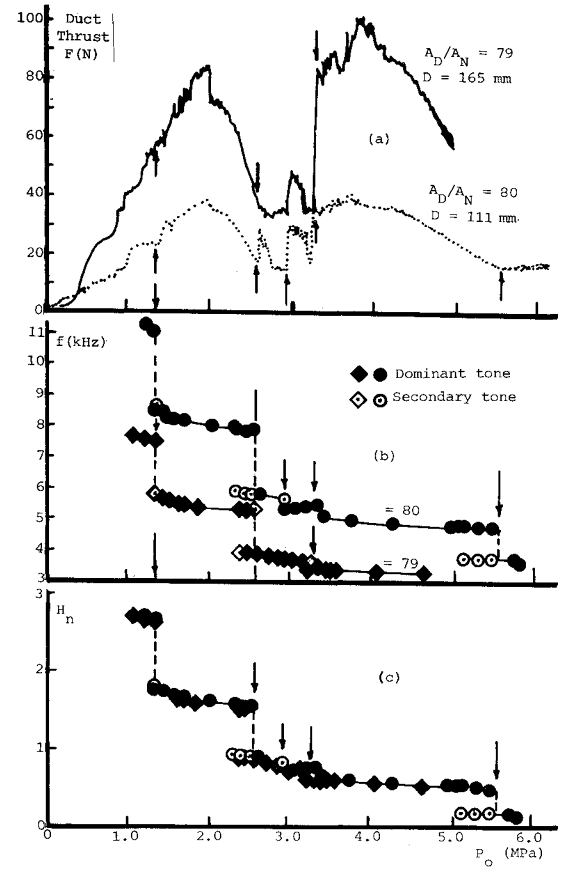

Fig. 5 Duct axial thrust and discrete tone fundamental frequency characteristics obtained for two ejectors of different duct and nozzle sizes but with the same A_D/A_N and L/D ratios. a) duct axial thrust, b) discrete frequency, c) Helmholtz number.

The experimental values of thrust augmentation ratio Φ obtained with the same ejectors and high pressure ranges used in the present investigation were previously reported by Fisher.[16] As is expected, the Φ characteristics showed similar irregularities with jet stagnation pressure ratio, and this is not surprising, as no irregularities were noticed in the primary nozzle thrust. Typical Φ values for the $A_D/A_N = 100$ and $L/D = 6.5$ ejector obtained from Ref. (16) are $\Phi = 1.54$, 1.38, 1.48 and 1.29, which correspond to $P_o/P_a = 20$, 32.3, 43.5, and 60, respectively.

Discrete Tone Emission

The irregularities displayed in the thrust characteristics shown in Figs. 2 and 3 were always accompanied by abrupt audible changes in the emitted noise. Spectral analysis of the noise radiated by each ejector configuration at various points on the characteristic was used to identify the fundamental frequency and the harmonics of the discrete tone. Typical examples of the noise spectra are shown in Fig. 4. In Fig. 4a, only one discrete frequency with its harmonics can be detected. By contrast, Fig. 4b features two discrete frequencies together with their harmonics; as will be shown later, this was always the case near the point on the frequency characteristic where the discrete frequency displayed a discontinuity in its variation with jet stagnation pressure ratio.

The duct axial thrust variation with jet stagnation pressure as traced by the X-Y Plotter for two ejectors of different size, but with the same L/D ratio and nearly the same A_D/A_N (79 for the first and 80 for the second) is shown in Fig. 5. As expected, the two thrust characteristics were qualitatively similar in most aspects with irregularities occurring almost at the same pressure. The variation of the discrete tone frequency with jet blowing pressure measured for these two ejectors is shown in Fig. 5b. From this figure, it is obvious that the discrete tone frequency was not a continuous function of jet-blowing pressure, but clear discontinuities occurred at certain pressures. It is also obvious that the occurrence of these frequency discontinuities coincided with the discontinuities in the thrust characteristics. In the narrow pressure range corresponding to these frequency discontinuities, the discrete tone frequency appeared to be multivalued and the upper and lower limits of the discontinuity overlapped. Two different discrete tones were detected from the noise spectra corresponding to this narrow range in similar fashion to those shown in Fig. 4b. The amplitudes of these fundamental discrete tones were different, and the one with the larger amplitude was called the dominant tone and the other secondary. The frequency discontinuity was assumed to occur at the jet stagnation pressure at which the two fundamental discrete tones had the same amplitude. The discrete tone fundamental frequencies measured for both ejectors are presented in Fig. 5c in terms of Helmholtz number (based on duct diameter) versus jet stagnation pressure. It is obvious that the tonal frequencies for both ejectors collapsed onto a single characteristic. This means that, for a given jet stagnation pressure, the emitted discrete tone frequency varies in inverse proportion to a characteristic length; it will be shown later that the relevant length is the ejector duct diameter.

The discrete tone fundamental frequency as a function of jet stagnation pressure ratio obtained for three ejector configurations of different duct diameter but with the same nozzle is shown in Fig. 6a. Shown in the same figure for comparison are the results obtained for same nozzle but without being confined in an ejector duct. It is obvious that the frequency characteristic for the bare nozzle appears to be more or less a continuously reducing function of jet stagnation pressure. When this nozzle was confined in an ejector duct, the measured discrete tone fundamental frequency appears to depart from the above continuous variation with stagnation pressure and to cluster around certain tonal frequencies which are different for differ-

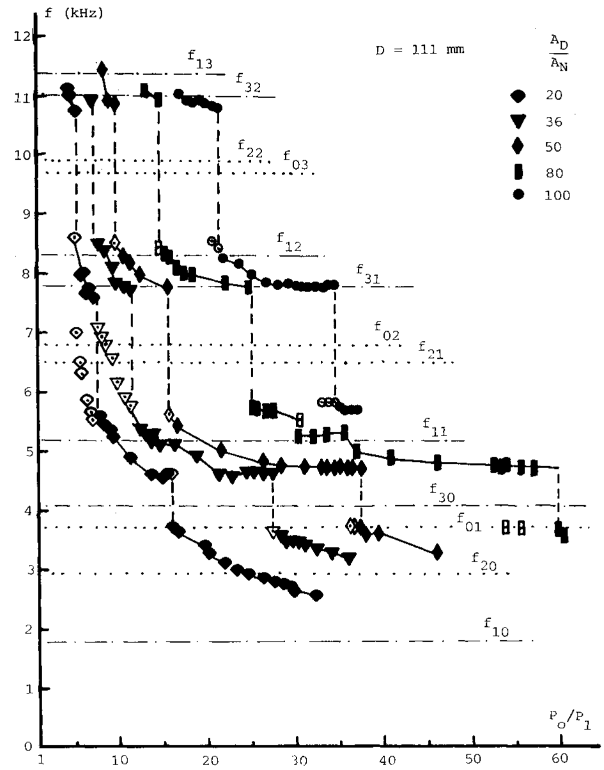

Fig. 7 Discrete tone fundamental frequency obtained for an ejector family of the same $D = 111$ mm but with different nozzle size. Closed and open symbols are for dominant and secondary tones respectively. . ., symmetric modes; —·—·—, Asymmetric modes; — — —, frequency discontinuities.

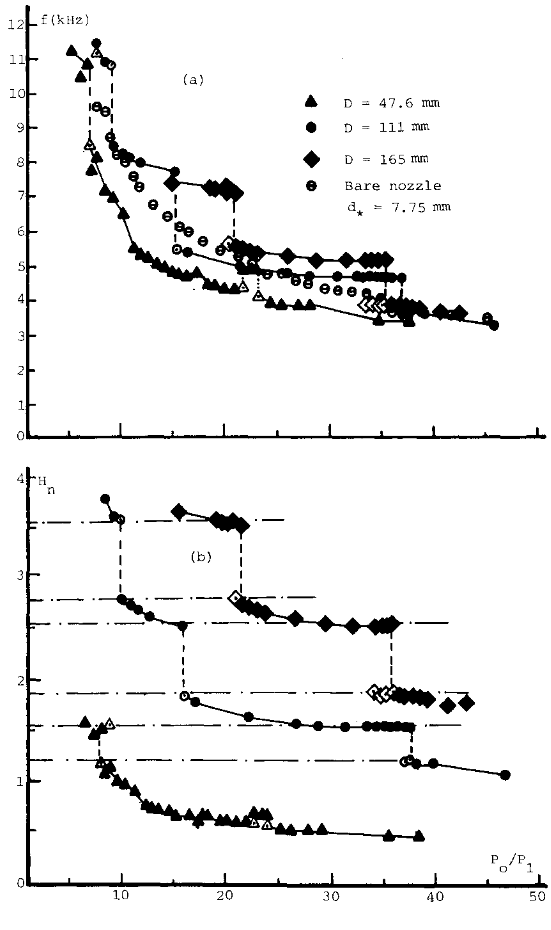

Fig. 6 Discrete tone fundamental frequency obtained with the same nozzle ($d_* = 7.75$ mm) for the free and confined jet configurations. a) frequency characteristics. b) Helmholtz number.

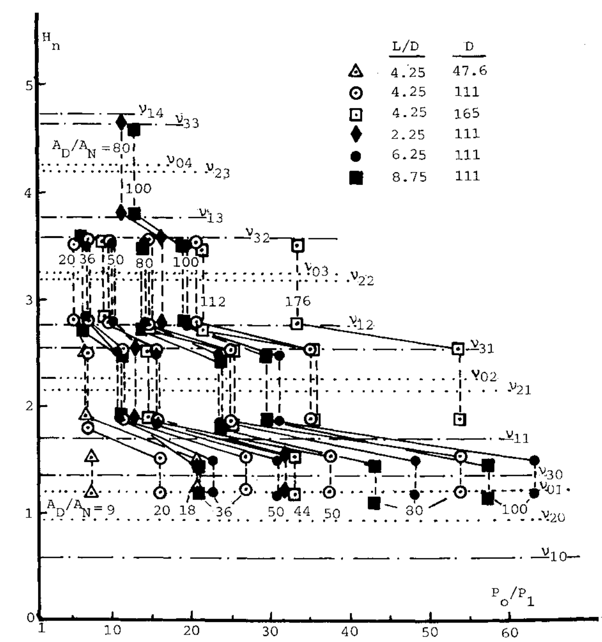

Fig. 8 Limits of frequency discontinuities for all ejector configurations tested in terms of H_n vs P_o/P_1.

ent duct diameters. For this reason, each of the three characteristics of the confined configurations is discontinuous and shows a staging behavior in its dependence on jet stagnation pressure ratio. Depending on the ejector configuration and the range of stagnation pressure, more than one discontinuity can be identified. The fact that the above bare nozzle characteristic was continuous would eliminate the possibility that the observed frequency discontinuities were inherent to the supply pipe duct modes (organ pipe modes) in particular or the supply system in general. The fundamental discrete tone characteristics obtained experimentally with the other four nozzles but in the unconfined environment, reported elsewhere,[25] were also continuous. These frequency discontinuities as shown in Fig. 6a appear to be unrelated, but when replotted in nondimensional terms as H_n vs P_o/P_1 in Fig. 6b, it is obvious that the Helmholtz number defining the upper or lower limits for each discontinuity range is the same for the three ejectors tested.

A plot of the discrete tone fundamental frequency vs jet stagnation pressure ratio for a family of ejectors of fixed duct diameter and length $D = 111$ mm and $L/D = 4.25$ but of different nozzle size is shown in Fig. 7. Similar plots, but not shown in this paper, were also obtained for two other ejector families, each of the same duct length to diameter ratio $L/D = 4.25$ but of different duct diameters ($D = 47.6$ mm and $D = 165$ mm) and nozzle size. For each family, the tonal frequencies which defined the limits of discontinuity ranges were nearly the same for each member of the family. Thus, frequency characteristics in the stages showed very weak dependence on jet stagnation pressure ratio, especially in the high-pressure range and with large duct to nozzle area ratios. However, the levels of jet stagnation pressure at which certain tones were generated and at which the discontinuous changes occurred for each family varied with nozzle size. A similar discrete tone staging behavior was reported by Hsia et al.[18] for the two-dimensional ejector configuration in a stagnation pressure range up to 4.4 atm. They found the discontinuous frequency ranges, or the step sizes between the stages, to be approximately equal. The results of the present investigations in Fig. 7 show the step size to be different for different stages, and for a given A_D/A_N ejector configuration the step size decreased with increasing stagnation pressure ratio.

The discrete tone frequencies presented in Fig. 7 are replotted in nondimensional terms as H_n vs P_o/P_1 in Fig. 8. For clarity the frequency characteristic in each stage is represented by a straight line. The results obtained for three other ejector families, each of the same duct diameter $D = 111$ mm, but of different L/D ratios ($L/D = 2.25$, 6.25, and 8.75), are also shown in Fig. 8. The occurrence of the frequency discontinuities obtained with the other two ejector families of fixed $L/D = 47.6$ mm and $D = 165$ mm) are also included in Fig. 8. This figure shows that the Helmholtz number which defined the limit of a given discontinuity is nearly the same for any A_D/A_N and L/D ejector configuration. However, the jet stagnation pressure that marked the occurrence of any frequency discontinuity varied with nozzle size and L/D ratio. From this it is clear that discrete tones were emitted only in certain ranges of Helmholtz number and were absent in others over the entire range of jet stagnation pressure and ejector configuration. It is also obvious that the Helmholtz number ranges, or the frequency bands where discrete tones were detectable, are much narrower than the discontinuous ones.

For a long circular duct, the theoretical transverse natural resonant modes can be obtained by solving the simple wave equation in cylindrical coordinates with the boundary conditions of finite perturbation amplitudes at the duct centerline and zero radial velocity at the duct wall. The solution is given by Rschevkin[19] as roots of derivatives of Bessel functions:

$$f_{p,n} = (c/\pi D)\nu_{p,n} \tag{1}$$

In this equation, each pair of p,n numbers defines a certain wave mode with oscillations in the direction perpendicular to

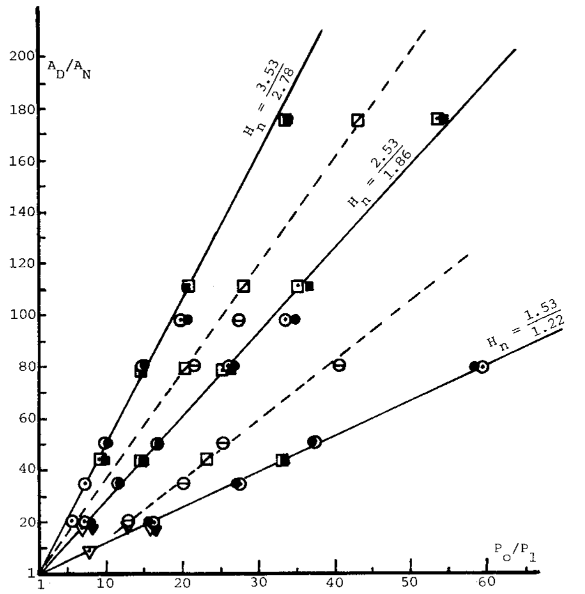

Fig. 9 Limits of frequency discontinuities obtained for different ejector configurations with $L/D = 4.25$. ▽, $D = 47.6$ mm; ⊙, $D = 111$ mm; ⊡, $D = 165.1$ mm; ▼, ●, ■, thrust characteristic irregularities; ▽, ⊖, ⊠, maximum duct thrust.

Table 1 Natural modes $f_{p,n}$ (kHz) for a long circular duct

Duct Diameter mm	p	n = 0	1	2	3	4
47.6		0	8.70	15.93	23.12	30.28
111.1	0	0	3.73	6.83	9.91	12.98
165.1		0	2.51	4.60	6.67	8.74
47.6		4.18	12.11	19.36	26.60	33.78
111.1	1	1.79	5.19	8.31	11.40	14.48
165.1		1.20	3.49	5.59	7.67	9.75
47.6		6.93	15.23	22.65	29.93	37.14
111.1	2	2.97	6.53	9.71	12.83	15.92
165.1		2.00	4.40	6.54	8.64	10.72
47.6		9.54	18.22	25.78	33.15	40.41
111.1	3	4.09	7.81	11.05	14.21	17.32
165.1		2.75	5.26	7.44	9.56	11.66

the circular duct axis, but with the wave mode propagated along the entire length of the duct without variation. In general, p and n specify the number of nodal diametral planes and internal nodal cylinders respectively along which both the acoustic pressure and the axial oscillation velocity are zero in the duct. The oscillation of the theoretical wave mode can be symmetric or asymmetric corresponding to p being an even or odd number respectively. In the asymmetric mode, the wave is propagated in the duct along a helical path, and this mode is sometimes called the spinning mode. This equation was used to calculate the first few symmetric and asymmetric transverse modes for the ducts used in the present investigation, and these are given in Table 1. These were calculated for the ideal case of a duct with no flow and uniform pressure and did not take into account the nonuniform flow structures through the ducts. These nonuniformities, which are primary jet dependent, include coflowing streams and shock cell formations associated with over or underexpansion of the supersonic primary jet in the ejector duct. These theoretical modes are compared with the experimental results in Fig. 7. In the range of discrete tone frequencies detected for the three families of ejectors all the theoretical aysmmetric modes (with the exception of f_{30}) agreed with the measured values of the apparent resonance and most of this coincidence occurred at the upper and lower limits of the frequency discontinuities separating the stages. On the other hand (with the exception of f_{01} and f_{20} in the low-frequency range) all the symmetric modes coincided with the discontinuous ranges between the stages where discrete tone frequencies could not be detected.

The Helmholtz number based on duct diameter and the natural duct resonance mode (H_n') is obtained by rewriting Eq. (1) as:

$$H_n' = f_{p,n}D/c = \nu_{p,n}/\pi \qquad (2)$$

which displays proportionality between H_n' and the respective $\nu_{p,n}$. Values of H_n' are compared with the measured Helmholtz number values in Fig. 8, and the agreements and disagreements are the same as for those of the f_{pn} values in Fig. 7.

The effect of the ejector duct length on duct thrust was clearly demonstrated in Fig. 3 and discussed in the previous section. Increased duct length, in addition to improving the levels of thrust augmentation in the relatively high-pressure ranges, also shifted thrust irregularities to lower pressure ranges for all ejector configurations tested.

Discrete tone frequencies were also measured for the other three ejector families of $L/D = 2.25$, 6.25 and 8.75. The duct diameter for these families was kept at the same fixed value ($D = 111$ mm) as that for the $L/D = 4.25$ family of ejectors. Although the irregularities in thrust characteristics tended to be less pronounced (Fig. 3), and those corresponding to lower stagnation pressure ranges tended to disappear with increasing duct length, discrete tone frequencies continued to be emitted displaying the same staging behavior observed with $L/D = 4.25$ ejectors. The measured tonal frequencies are not included in this paper, but the occurrence of their frequency discontinuities are included in Fig. 8. This figure shows that the Helmholtz numbers corresponding to the upper or lower limits of a given discontinuity were nearly invariant, but the jet pressures at which these discontinuities occur varied with duct length in addition to nozzle size. For an ejector duct of circular cross section, Fig. 8 clearly indicates that discrete tone frequencies are only emitted in certain bands of Helmholtz number. These bands can be theoretically predicted and are the same for any circular duct of any diameter and length, but the jet pressure corresponding to the occurrence of the discontinuities varies with nozzle size and duct length.

The occurrence of each frequency discontinuity in Fig. 7 for the three ejector families of different duct diameter but with the same $L/D = 4.25$ is replotted as A_D/A_N vs jet stagnation pressure ratio P_o/P_1 in Fig. 9. From this it is obvious that each of the discontinuity bands assumed a linear variation with jet stagnation pressure ratio and each can be ascribed two different but fixed values of H_n, the larger of which corresponds to the upper limit of the discontinuity and the smaller to the lower limit. These H_n values roughly correspond to the theoretical values based on duct diameter and natural resonance modes shown in Fig. 8.

The jet stagnation pressure ratio corresponding to the occurrence of the irregularities in thrust characteristics are also shown in Fig. 9, and most of these appear to coincide with the corresponding frequency discontinuities and to assume the same linear variation. Also shown in Fig. 9 is the occurrence of the maxima on the thrust characteristics. Although these maxima were difficult to determine in the high pressure ranges and at large A_D/A_N ratios, they appeared to assume a similar linear variation but with different proportionality constants.

As was the case with $L/D = 4.25$ ejectors, the discrete tone frequency bands for the $L/D = 6.25$ and 8.75 ejectors also assumed linear variation with jet stagnation pressure as shown in Figs. 10a and 10b, respectively. The corresponding thrust irregularities, whenever they can be detected, are also shown in Figs. 10a and 10b, and they appeared to vary linearly with jet stagnation pressure ratio. The relationship that defines this linear variation for a given frequency discontinuity band was found to be different for different L/D ratios, as is clearly illustrated in Fig. 11 for the four ejector families. From this figure it is obvious that the slope or the linear proportionality constant increased with increasing ejector duct length, accompanied by a slight reduction in the Helmholtz number corresponding to a given frequency discontinuity band. It also illustrates the fact that, for a given A_D/A_N, the effect of duct length on the P_o/P_1 required for the occurrence of a frequency discontinuity becomes less pronounced in the lower ranges of P_o/P_1 or the higher ranges of H_n.

Flow Visualization

To reveal the effect of duct resonance on the confined jet mixing and the entrainment process, schlieren photographs were taken with a plane walled duct which was basically square in cross section with generous corner fillets added. The duct was equivalent in cross-sectional area and length to the $A_D/A_N = 36$, $L/D = 4.25$ axisymmetric configuration. Although its resonance characteristics were somewhat more complex, it is believed that their effects on the mixing process and flow phenomena involved were at least qualitatively similar.

The two photographs presented in Fig. 12 reveal some features of the duct flowfield in two ranges of jet stagnation pressure. The first was in a pressure range close to the peak of the thrust characteristic or during a resonant condition and the other in a higher pressure range outside the upper boundary of duct resonance. In the second photograph, the jet appeared to be relatively stable and mixing retarded. In the first photograph, the jet appeared to be oscillating and unstable and surrounded by interspersed eddy or vortex formations which occurred alternately on opposite sides of the jet along its length. In addition to the Mach wave radiation usually associated with supersonic jet flows, the portion of the jet immediately downstream or the nozzle appeared to be surrounded by distinct clusters of wavelets or weak shock waves. These waves

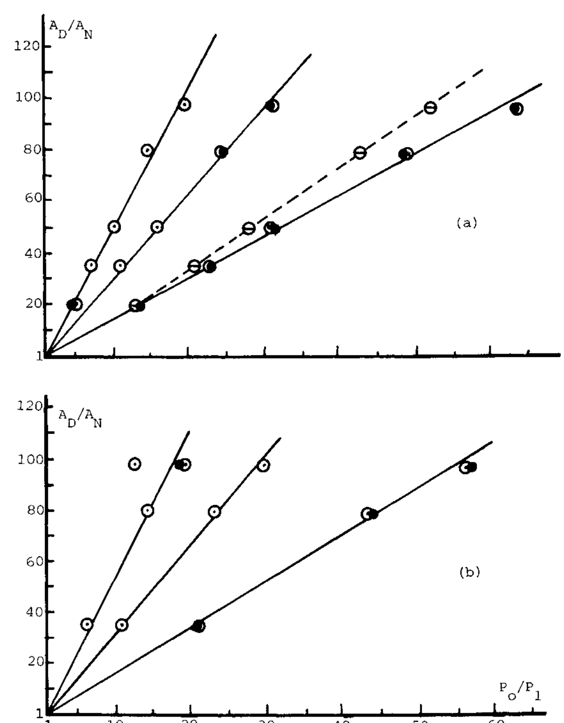

Fig. 10 Limits of frequency discontinuities obtained for two families of ejectors. a) $L/D = 6.25$; b) $L/D = 8.75$: ⊙, frequency discontinuities; ⊖, maximum duct thrust; ●, thrust characteristic irregularities.

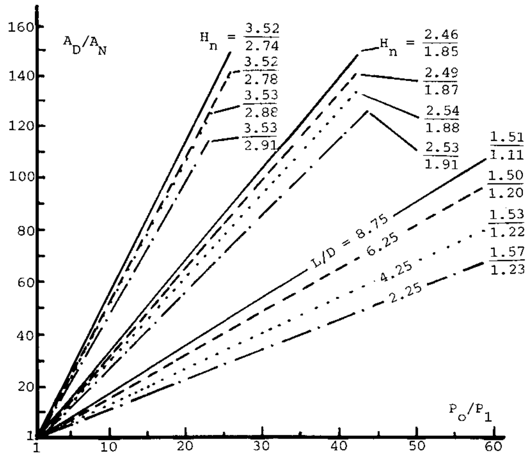

Fig. 11 The linear variation of three frequency discontinuity bands for four ejector families of the same duct diameter but with different *L/D* ratios.

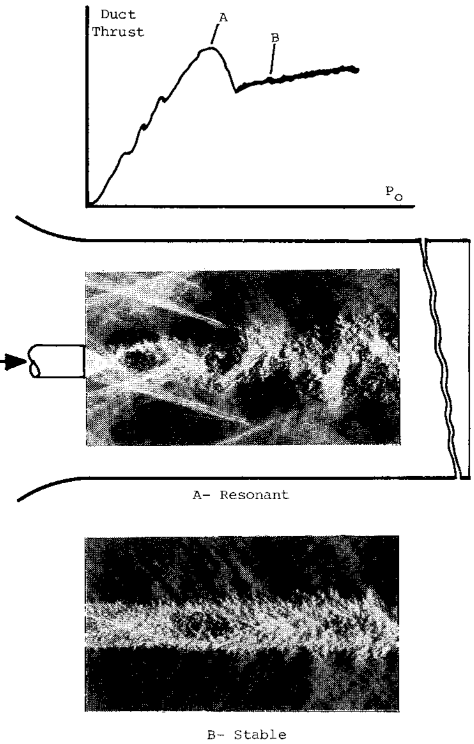

Fig. 12 Schlieren photographs of the axisymmetric jet in the square duct during resonant and relatively stable flow conditions corresponding to points A & B on the thrust characteristic respectively.

were found to propagate in the upstream direction, and this was confirmed with the aid of photographs taken to include the bell mouth and the upstream flowfield. They are thought to be generated from the intense sound waves emitted from the screech sources at the shock cell end locations in the shear layer along the jet boundary.[3,4] Very well defined arcs of sound waves were observed in some of the photographs with centers coinciding with the shock cell extremities in the shear layer or very close to it. It is believed that the bands or clusters of weak shock waves (strong Mach waves) were formed by the coalescence of waves resulting from the regular or irregular interaction between the emitted sound waves (screech) and the solid walls of the duct. Due to the fact that radiation of sound waves from the source in the jet boundary is always spherical, the angle of incidence between these waves and the duct plane solid walls varies with time as the spherical wave progress away from the source. The orientation relative to the jet axis and the strength of the reflected waves will therefore vary with time, and what appears in the photographs will depend on the instant the photograph was taken. In depth study of the exact nature and behavior of these weak shock waves is underway and will be reported in the future. Waves of nearly similar behavior were observed by Sorohia[20] during his flight simulation experiments with incorrectly expanded supersonic jets from sonic nozzles with imposed coflowing streams but without the solid duct boundaries. He concluded that these waves were generated at random from the region of the flowfield downstream of the nozzle where the jet was completely subsonic, and were propagated in the upstream direction towards the nozzle.

With the aid of a large number of similar photographs taken simultaneously in two perpendicular planes it was possible to establish that in the resonant condition the jet adopted a helical instability mode, evidently driven by the pressure waves reflected from the duct walls. This helical instability mode caused the jet to show large spinning oscillations which appeared in the photographs to start just downstream of the second shock cell. These oscillations were associated with marked distortion of the cellular shock structure, enhanced spreading of the jet and its mixing with the entrained coflowing secondary stream, and marked increase in the level of entrainment. This spinning motion tended to shift the concentration of the energy from the duct axis to an intermediate radius, and is believed to be responsible for the unusual saddle shaped mean velocity profiles reported in Refs. (18, 22–24) at the enhanced mixing condition. Mixing in turbulent shear flows is attributed to large scale turbulent structures such as vortices or eddies.[21,22] During this helical instability mode, the jet always appeared to be surrounded by interspersed helical eddies, which seemed to grow as they progressed downstream. This led to distortion of the shock cell structure in the jet core and subsequent enhancement of jet spreading and decay.

As stated earlier, irregularities in thrust characteristics were always associated with discontinuities in discrete tone emission. The effect of discrete tones on mixing and ejector performance is therefore thought to be due to their effect on the large-scale turbulent eddies in the shear layer. For a free incorrectly expanded supersonic jet, the link is readily available through Powell's freedback mechanism,[3,4] where, for a fixed shock cell length, the discrete tone wave length is directly related to the eddy convection velocity or frequency. For a given free jet stagnation pressure ratio, the shock cell length is fixed and can be determined easily.[25] For a confined jet at the resonant condition, the discrete tone frequency tends to cluster around one of the duct natural resonant modes corresponding to a range of jet stagnation pressure ratio; hence the staging behavior. Sound waves emitted from the sources along the jet boundary are intensified through reflections from the solid duct walls and subsequent coalescence to form bands of weak shock waves. In their passage upstream, these waves interact obliquely with the jet resulting in some changes in the jet behavior and distortions to the cellular shock structure before their arrival at the nozzle lip to produce the disturbance as required in Powell's feedback mechanism for discrete tone emission from incorrectly expanded supersonic free jets. This added complication of shock cell distortion made it difficult to perform reliable shock cell length measurements in the vicinity of the sound sources downstream, and therefore a reasonable quantitative verification to the feedback mechanism for the confined supersonic jet is not available at this stage.

IV. Conclusions

The main conclusions of this investigation are as follows:

1) The thrust characteristics obtained experimentally with changing jet stagnation pressure ratio for all ejector configurations tested were found to display irregularities which could

not be predicted or explained by the inviscid one-dimensional ejector theory. These were found to be more pronounced with relatively short ejectors and large duct to nozzle area ratios.

2) Spectral analysis of the emitted ejector noise resulted in the following observations:

a) Discrete tones of acoustic radiation of varying amplitude were always emitted from every ejector configuration tested at any jet stagnation pressure ratio.

b) For a given duct to nozzle area ratio, the discrete tone fundamental frequency exhibited staging behavior in its variation with jet stagnation pressure.

c) For small duct to nozzle area ratios, or in the relatively low range of jet stagnation pressure, the discrete tone frequency in the stages was a continuously reducing function of jet stagnation pressure. On the other hand, it showed very weak dependence on jet stagnation pressure in the high-pressure ranges and with large duct to nozzle area ratios.

d) The jet stagnation pressure ratios corresponding to the frequency discontinuities separating the stages were found to coincide with those at which irregularities in thrust characteristics occurred. The tonal frequencies at these discontinuities were found to coincide with the theoretical symmetric and asymmetric natural modes of the mixing duct in the low- and high-frequency ranges, respectively. The resonant modes can therefore be correlated with gross changes observed in ejector performance; this provides a clue as to why these changes occur, but it does not explain how the performance was affected.

e) For a given band of frequency discontinuity, the Helmholtz number ($H_n = fD/c$) was found to be the same for every ejector configuration tested.

f) For each family of ejectors having different duct and nozzle sizes but the same duct length to diameter ratio the jet stagnation pressure at which a certain discontinuity occurred was found to vary linearly with duct to nozzle area ratio. The irregularities in thrust characteristics were found to assume similar linear relationships. The proportionality constant for this linear variation increased with duct length.

3) Increasing the duct length for a given value of duct to nozzle area ratio greatly improved the levels of thrust augmentation in the high jet stagnation pressure range and shifted the irregularities in thrust characteristics and the corresponding tonal frequency discontinuities to lower pressure ranges.

4) From the single- and two-plane schlieren photographs of the duct flow field taken at the resonant condition, the following observations were made:

a) There was a marked increase in the effective spread rate of the jet.

b) Bands of weak shock waves were present in the subsonic duct flow field and appeared to propagate in the upstream direction toward the nozzle. These are thought to result from coalescence of the reflected Mach waves from the duct walls.

c) The jet appeared to be surrounded by interspersed helical eddies which grew in size during their passage downstream.

d) The jet appeared to be affected by a helical or spinning instability mode forcing the jet to oscillate along a helical path thereby shifting the concentration of energy from the center of the duct to an intermediate radius.

References

[1]Brown, G. B., "On Vortex Motion in Gaseous Jets and the Origin of their Sensitivity to Sound," *Proceedings of the Physical Society*, London, Vol. 47, 1935, pp. 703–732.

[2]Hammitt, A. G., "The Oscillation and Noise of an Over Pressure Sonic Jet," *Journal of the Aerospace Sciences*, Vol. 28, Sept. 1961, pp. 673–680.

[3]Powell, A., "On the Noise Emanating From a Two-Dimensional Jet Above the Critical Pressure," *The Aero. Quarterly*, Vol. IV, Feb. 1953, pp. 103–122.

[4]Powell, A., "On the Mechanism of Choked Jet Noise," *Proceedings of the Physical Society*, London, Sec. B, Vol. 66, 1953, pp. 1039–1056.

[5]Glass, D. R., "Effect of Acoustic Feedback on the Speed and Decay of Supersonic Jets," *AIAA Journal*, Vol. 6, Oct. 1968, pp. 1890–1897.

[6]Nagel, R. T., Denham, J. W., and Papathanasiou, A. G., "Supersonic Jet Screech Tone Cancellation," *AIAA Journal*, Vol. 21, Nov. 1983, pp. 1541–1545.

[7]Poldervaart, L. J., Wijnands, A. P. J., and Bronkhorst, L., "Aerosonic Games With the Aid of Control Elements and Externally Generated Pulses," AGARD-CP-131, Sept. 1973, pp. 20-1—20-4.

[8]Ali, M. R. M. and Anderson, J. S., "Noise Generated by Air Flow Through Circular Nozzle of Different Exit Areas," *Proceedings of the Seventh International Congress on Acoustics*, Budapest, Hungary, Paper No. 25N14, 1971, pp. 536–540.

[9]Middleton, D. and Richards, E. J., "Discrete Frequency Noise from Ejector Nozzles," *Proceedings of the Fourth International Congress on Acoustics*, Copenhagen, Denmark, Paper No. L21, Aug. 1962.

[10]Middleton, D., "The Noise of Ejectors," Aeronautical Research Council, London, Research and Memoranda 3389, 1965.

[11]Reid, J., "The Effect of the Cylindrical Shroud on the Performance of a Stationary Convergent Nozzle," Aeronautical Research Council, London, Research and Memoranda 3320, 1963.

[12]Bornd, P. S., "Performance and Noise Generation Studies of Supersonic Air Ejectors," *Proceedings of the Seventh International Congress on Acoustics*, Budapest, Hungary, Paper No. 24N15, 1971, pp. 481–483.

[13]Mamin, V., Pykhov, Z., and Rimski-Korsckov, A., "Discrete Tone Radiation Arising from a Supersonic Jet Flowing into Unlimited Gaseous Medium and into a Cylindrical Ejector," *Proceedings of the Seventh International Congress on Acoustics*, Budapest, Hungary, Paper No. 24N11, 1971, pp. 469–472.

[14]Quinn, B., "Effect of Aeroacoustic Interactions on Ejector Performance," *Journal of Aircraft*, Vol. 23, Nov. 1975, pp. 914–916.

[15]Quinn, B., "Interactions Between Screech Tones and Ejector Performance," *Journal of Aircraft*, Vol. 14, May 1977, pp. 467–473.

[16]Fisher, S. A., "Thrust Augmenting Ejectors for High Pressure Ratio Propulsive Jets," Seventh Australasian Hydraulics and Fluid Mechanics Conference, Brisbane, Australia, Aug. 1980, pp. 246–249.

[17]Abdel-Fattah, A. M., "Duct Resonance in High Pressure Ratio Thrust Augmenting Ejectors," Proceedings of the Second Workshop On Wind Engineering and Industrial Aerodynamics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1985.

[18]Hsia, Y., Krothapalli, A., Baganoff, D., and Karamcheti, K., "An Experimental Investigation of an Underexpanded Rectangular Jet Ejector," Joint Institute for Aeronautics and Acoustics, Stanford Univ., Stanford, CA, TR-53, April 1984.

[19]Rschevkin, S. N., "A Course of Lectures on the Theory of Sound," translated from Russian by O. M. Blunn, Pergamon Press, 1963, p. 188.

[20]Sarohia, V., "Some Flight Simulation Experiments on Jet Noise From Supersonic Underexpanded Flows," *AIAA Journal*, Vol. 16, July 1978, pp. 710–716.

[21]Brown, G. L. and Roshko, A. A., "On the Density Effects and Large Structure in Turbulent Mixing Layers," *Journal of Fluid Mechanics*, Vol. 64, July 1974, pp. 775–816.

[22]Yule, A. J., "Large-Scale Structure in the Mixing Layer of a Round Jet," *Journal of Fluid Mechanics*, Vol. 89, Dec. 1978, pp. 413–432.

[23]Quinn, B., and Toms, H. L., "Mixing Duct Pressure Distributions and Exhaust Flow Characteristics of a High Temperature and Pressure Cylindrical Ejector," Aerospace Research Lab., Wright-Patterson Air Force Base, OH, Rept. No. 75-0227, 1975.

[24]Fisher, S. A. and Abdel-Fattah, A. M., "Flow Measurements in a High Pressure Cylindrical Ejector," Ninth Australasian Fluid Mechanics Conference, Auckland, New Zealand, Dec. 1986.

[25]Abdel-Fattah, A. M., "Discrete Tone Emission From High Pressure Ratio Supersonic Jets from Convergent Divergent Nozzle," *AIAA Journal* (submitted for publication).